INEQUALITIES AND REGIONS

Summary:

1. *The following symbols are used when dealing with inequalities* $<$, $\leq$, $>$ *and* $\geq$

2. *The inequality symbol reverses when you multiply or divide an inequality by a negative number*

3. *To represent an inequality on a number line, use an open circle for* $<$ **or** $>$ *symbol and in case of* $\leq$ *or* $\geq$*, use a closed circle.*

4. *Integers in the range of a given inequality are called integral values*

5. *The inequality* $2 > x$ *is the same as* $x < 2$ *and* $4 < x$ *is the same as* $x > 4$.

EXAMPLES:

1. *Represent each of the following inequalities on a number line:*

(i) $x \geq -2$ (ii) $x > 3$ (iii) $x \leq 2$ (iv) $x < 4$ (v) $-1 \leq x \leq 5$

(vi) $-2 < x < 3$ (vii) $-1 < x \leq 4$ (viii) $-3 \leq x < 2$

2. *Given that* $P = \{x : -3 \leq x < 4\}$ *and* $Q = \{x : -2 < x \leq 6\}$, *represent* **P n Q** *on a number line. State* **P n Q**

3. *Solve the following inequalities and represent each solution on a number line:*

(i) $5x + 7 < 3(x + 1)$ (ii) $7(2 - x) + 1 \leq 2(2x - 9)$ (iii) $5x + 3 > -11 - 2x$

(iv) $3(x - 1) + 2(x - 1) \leq 7x + 7$ (v) $\frac{3}{2} - \frac{5x}{3} > 8 + \frac{x}{2}$ (vi) $\frac{x}{4} + 3 \geq 1 + \frac{x}{2}$

(vii) $\frac{3x}{2} - \frac{2}{3}(1 - 2x) < 5$ (ix) $7 \geq 4 - 3x > -5$ (x) $2x - 4 \leq 4 > -3x - 5$

4. *Using a number line, find the integral values of* **x** *which satisfy the sets:*

$\{5 - 3x > -7\}$ **n** $\{x - 6 \leq 3x - 4\}$

5. *Find all the integral values of* $\boldsymbol{x}$ *which satisfy the inequalities:*

$$\frac{5x+7}{4} \le \frac{3x+5}{2} < \frac{x+11}{3}$$

6. *Find the positive integral values of* $\boldsymbol{x}$ *which satisfy the inequalities:*

$$\frac{x}{4} - 3 \le x + 2 \le 21 - 2x$$

7. *Find the greatest integral value of* $\boldsymbol{x}$ *which satisfies the inequality:*

$$2 - \frac{3x}{2} > x + 3$$

8. *Given that* $-1 < x < 4$*,find the values of* $\boldsymbol{a}$ *and* $\boldsymbol{b}$ *for which* $a \le 2x + 3 < b$

EER:

1. *Solve the inequality:* $\frac{x}{4} + 5 \ge 1 + \frac{x}{2}$

2. *Solve the inequality:* $10x - 3(2x-1) \ge 8x + 15$

3. *Solve the inequality:* $\frac{2x-3}{5} \ge \frac{x}{2} - 1$

4. *Solve the inequality:* $3(x-2) + 4 \le 2(2x-3)$

5. *Solve the inequality:* $-6 \le 2(x-5) < 4$

6. *Solve the inequality:* $-3 < \frac{3}{2}(2-x) \le 5$

7. *Using a number line, find the integral values of* $\boldsymbol{x}$ *which satisfy the sets:*

$$\{3x > 2x + 5\} \text{ n } \{3x < 32 - x\}$$

8. *Solve the inequality:* $\frac{1}{2} - \frac{x}{6} > -\frac{5}{2}$

9. *Find the range of values of* $\boldsymbol{x}$ *which satisfy the inequalities:*

$x - 4 \le 3x + 2 < 2(x + 5)$

10. *Given that* $\boldsymbol{P = \{x:\ -4 \le x \le 2\}}$ *and* $\boldsymbol{Q = \{x:\ -2 < x < 5\}}$***,*** *represent* ***P n Q*** *on a number line****.*** *State* ***P n Q***

11. *Solve the inequality****:*** $\boldsymbol{\frac{3}{2} - \frac{5x}{3} > 8 + \frac{x}{2}}$

12. *Find all the integral values of* ***x*** *which satisfy the inequalities****:***
$\boldsymbol{2x + 3 \ge 5x - 3 > -8}$

13. *Find all the integral values of* ***x*** *which satisfy the inequalities****:***
$\boldsymbol{2x - 4 \le 4 > -3x - 5}$

GRAPHING LINEAR INEQUALITIES

Summary:

1. *In shading out the unwanted region****,*** *we proceed as follows****:***

(i) *Make* ***y*** *the subject in the given inequality equation*

(ii) *Rewrite the equation in the form* ***y = mx + c***

(iii) *Draw a solid line if the inequality is* $\le$ *or* $\ge$ *and in case the inequality is*
< or >, *draw a dotted line*

(iv) *If the inequality in* ***(i)*** *above is* ***> or ≥ ,*** *the wanted region is above the line and If the inequality is* ***< or ≤ ,*** *the wanted region is below the line****.*** *Thus we shade out the unwanted region*

2. *The points* ***(x, y)*** *within and on the boundary of the wanted region are called an integral solution* ***(x*** *and* ***y*** *are integers****)***

3. *The maximum or minimum value of any function in the wanted region occurs at one of its vertices*

EXAMPLES:

1. *Given that* $\boldsymbol{P = \{(x, y):\ 2x - 3y \le 6\}}$ *and* $\boldsymbol{Q = \{(x, y):\ x + y < 0\}}$***,*** *by shading the unwanted region****,*** *show the region representing* ***P n Q***

2. (i) *By shading the unwanted region, show the region representing*

$$\{(x, y):\ y \geq 6 - x \text{ n } y - x > 0 \text{ n } y \leq 7\}$$

(ii) *Find the integral solution of the inequalities*

(iii) *Calculate the area of the wanted region*

3. (i) *By shading the unwanted region, show the region which satisfies the inequalities:* $3x + 4y < 12$, $y \geq 0$ *and* $x \geq 0$

(ii) *Find the integral solution of the inequalities*

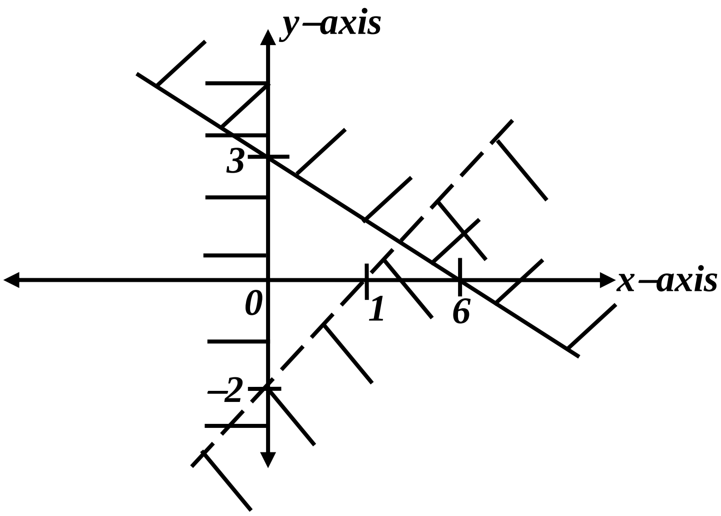
(ii) *Calculate the area of the wanted region*

4. *The feasible region of a linear inequality problem is represented by:*

$2 \leq x \leq 6,\ 1 \leq y \leq 5 \leq 6$ *and* $x + y \leq 8$

(i) *Draw the feasible region that represents this problem*
(ii) *Find the maximum value of the function* $f(x, y) = 3x + 2y$ *on the feasible region*
(iii) *Calculate the area of the feasible region*

5. (i) *Find the inequalities satisfied by the unshaded region below:*



(ii) *Calculate the area of the unshaded region*

6. ***(i)*** *By shading the unwanted regions, show clearly the region **R** which satisfies the inequalities:* $y - x < 2$, $2y + 5x \leq 25$ *and* $6y + x \geq 5$

(ii) *Given that* $P(x, y) = 50x + 40y$, *determine the maximum and minimum values of **P** in the region **R**.*

(iii) *Determine the area of the unshaded region **R***

7. *By shading the unwanted region, show the region representing* $y > x^2$ *for* $-2 \leq x \leq 2$

8. *By shading the unwanted region, show the region representing* $y > x^2 - 1$ *for* $-2 \leq x \leq 2$

EER:

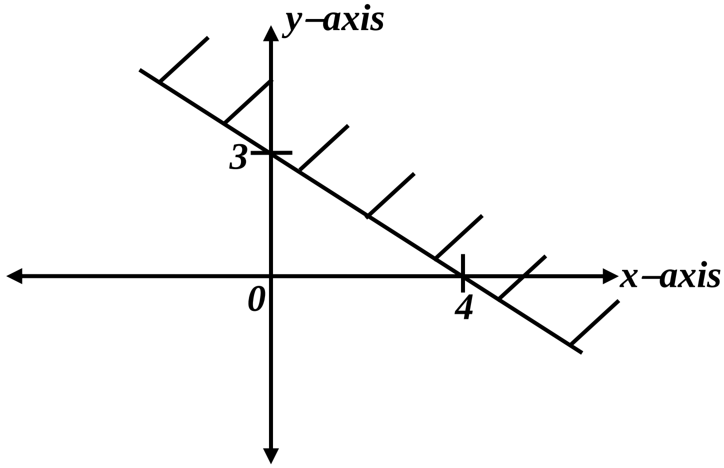
1. *By shading the unwanted region, show the region which satisfies the inequality* $3x + 4y < 12$

2. ***(i)*** *By shading the unwanted region, show the region representing*

$$\{(x, y): y \geq x - 2 \ \boldsymbol{n} \ y + x \leq 14 \ \boldsymbol{n} \ y \leq 7x - 26\}$$

(ii) *Calculate the area of the wanted region*

3. *Find the inequality that satisfies the unshaded region below:*



4. ***(i)*** *By shading the unwanted region, show the region which satisfies the inequalities:* $y \le x + 2$, $1 \le x \le 4$ *and* $y \ge 1$

(ii) *Calculate the area of the wanted region*

5. *Find the minimum and maximum values of the function* $f(x, y) = 40x + 15y$, *subject to the following constraints:*

$$5x + 4y > 40, \quad x \ge 2y, \quad 8x + 3y \le 90 \text{ and } y \ge 0$$

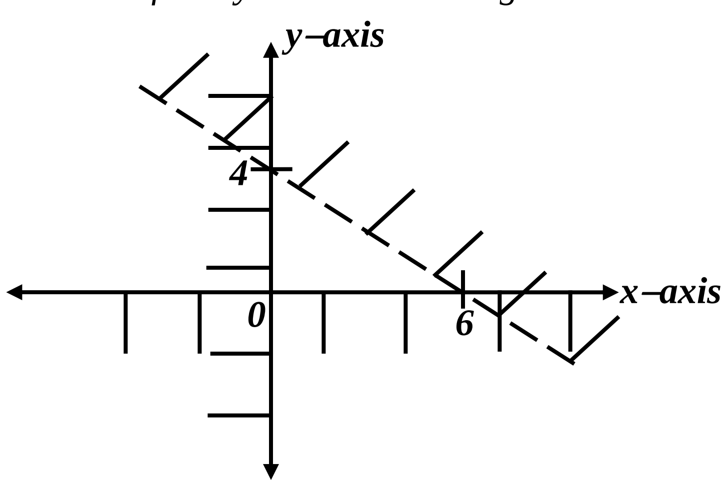
[Ans: min = 285** occurs at **(5, 5), max = 445** occurs at **(10, 3),]

6. ***(i)*** *By shading the unwanted region, show the region which satisfies the inequalities:* $x \le 4$, $2y + x \ge 4$ *and* $4y - 3x \le 8$

(ii) *Find the integral solution of the inequalities*

(iii) *Find the maximum and minimum values of* $P = x + y$ *in the wanted region.*

7. *Find the inequalities satisfied by the unshaded region below:*



8. ***(i)*** *By shading the unwanted region, show the region representing*

$$\{(x, y):\ y \ge 1 \ \mathbf{n}\ y + x \le 5 \ \mathbf{n}\ x \ge 1\}$$

(ii) *Calculate the area of the wanted region*

9. (i) *On the same axes, draw the curve* $y = 4 - x^2$ *for* $-2 \leq x \leq 2$ *and the line* $y = 1$

(ii) *By shading the unwanted region, show the region represented by* $y \leq 4 - x^2$ *and* $y \geq 1$

(iii) *State the integral coordinates of the points which lie in the region*

$$\left\{ y \geq 1 \cap y \leq 4 - x^2 \right\}$$

QUADRATIC INEQUALITIES

Summary:

1. *Solving a quadratic inequality is the same as find the range of* $x-$*values where the graph in the equation will be above or below the x–axis*

2. *The following steps apply when solving a quadratic inequality:*

(i) *Replace the original inequality with a quadratic equation*

(ii) *Solve the equation to get the endpoints of the three different intervals*

(iii) *Plot the solution on a number line to identify the intervals for investigation*

(iv) *Pick a number from each interval and work out the sign for each interval*

(v) *The symbol in the inequality determines the required range. In any interval the graph is either above or below the x–axis*

EXAMPLES:

1. *Find the range of* x *for which* $x^2 + x - 12 \leq 0$

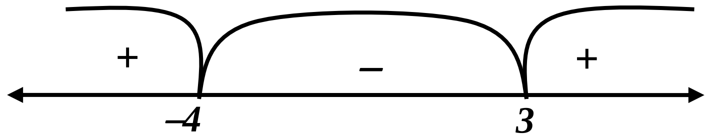
Soln:

At the endpoints, $x^2 + x - 12 = 0$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 48}}{2}$$

$\therefore \quad x = -4$ **or 3**

Testing for negativity (negative sign)



Required range = $-4 \le x \le 3$

NOTE: *The final answer must have the symbol used in the original inequality*

2. *Solve for* **x** *in the inequality:* $x^2 - x - 6 > 0$

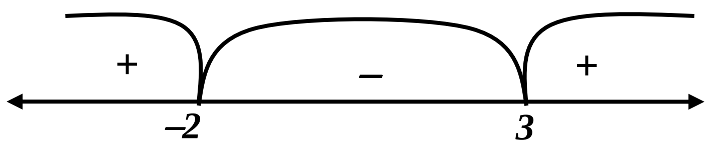
Soln:

At the endpoints, $x^2 +- x - 6 = 0$

$$\Rightarrow \; x = \frac{1 \pm \sqrt{1 + 24}}{2}$$

$\therefore \quad x = -2 \text{ or } 3$

Testing for positivity



Required range = $x < -2$ **or** $x > 3$

3. *Solve for* **x** *in the inequality:* $x^2 - 36 < 0$

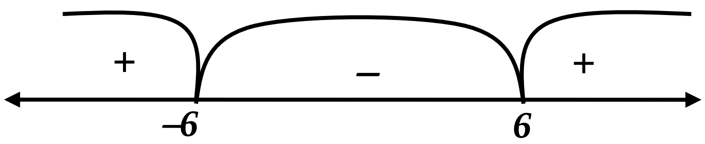
Soln:

At the endpoints, $x^2 - 36 = 0$

$$\Rightarrow \; x = \pm\sqrt{36}$$

$\therefore \quad x = -6 \text{ or } 6$

Testing for negativity



Required range = $\boldsymbol{-6 < x < 6}$

EER:

1. *Solve for* **x** *in the inequality:* $\boldsymbol{x^2 - 4x + 3 < 0}$

2. *Solve for* **x** *in the inequality:* $\boldsymbol{x^2 + 2x - 15 \geq 0}$

3. *Solve for* **x** *in the inequality:* $\boldsymbol{(x+2)(x-4) < x^2 - 6}$

4. *Solve for* **x** *in the inequality:* $\boldsymbol{2x^2 + 4x \geq x^2 + 5x + 6}$

5. *Determine the solution set of the inequality:* $\boldsymbol{4x^2 - 5x - 6 < 0}$

6. *Find the integral values of* **x** *which satisfy the inequality:* $\boldsymbol{2x^2 + 5x - 3 < 0}$

LINEAR PROGRAMMING

Summary:

1. *Linear programming provides the most productive decision that best fits the situation. It reveals the best criteria of distributing limited resources to achieve a desired objective. This objective may be profit maximization or cost minimization*

2. *In any linear programming problem:*
(i) *Alternative decisions are compared using an objective function to get the best.*
(ii) *The objective function takes the form* ***f(x, y) = ax + by*** *and its maximum* ***or*** *minimum value occurs at one of the vertices of the wanted region (feasible region)*
(iii) *If two corner points produce the same maximum* ***or*** *minimum value of the objective function, then every point on the line segment joining these points will also give the same maximum* ***or*** *minimum value*
(v) *The points within and on the boundary of the feasible region are called feasible solutions. These points* ***(x, y)*** *are non negative. Thus* $\boldsymbol{x \geq 0}$ *and* $\boldsymbol{y \geq 0}$

3. *Linear programming problems are solved as follows***:**

(i) *Define the variables* ***x*** *and* ***y***

(iv) *Write an expression to be maximized or minimized* **(***objective function***)**

(ii) *Find the inequalities including the non negative restrictions* $x \geq 0$ *and* $y \geq 0$

(iii) *Graph the inequalities and locate the vertices of the feasible region*

(v) *Substitute values from the vertices into the function and select the greatest or least result*

4. *The table below shows inequality symbols used for specific phrases*

Symbols	***Vocabulary***
$<$	**(i)** *Less than* **(ii)** *Fewer than* **(iii)** *Lower than* **(iv)** *Smaller than* **(v)** *Shorter than* **(vi)** *Below*
$>$	**(i)** *Greater than* **(ii)** *More than* **(iii)** *Exceeds* **(iv)** *Larger than* **(v)** *Longer than* **(vi)** *Above*
$\leq$	**(i)** *Less than or equal to* **(ii)** *At most* **(iii)** *Maximum* **(iv)** *Not more than* **(v)** *Not greater than* **(vi)** *Does not exceed* **(vii)** *Not above*
$\geq$	**(i)** *Greater than or equal to* **(ii)** *At least* **(iii)** *Minimum* **(iv)** *Not less than* **(v)** *Not fewer than* **(vi)** *Not below* **(vii)** *Not smaller than*

5. *When interpreting inequality word problems**,** identify the inequality symbol that is appropriate for the situation*

EXAMPLES:

1. *Write down the following restrictions in terms of algebraic inequalities*
(i) *There must be at least thrice as many **x** as **y***
(ii) *There must be at most **4** times as many **x** as **y***
(iii) *At least two–fifth of **(x + y)** should be **x***
(iv) *The value of **x** lies between **4** and **7***
(v) *The value of **x** lies between **4** and **7** inclusive*
(vi) *The value of **x** is at least **3** but not more than **6***
(vii) *The value of **x** is at least **3** but less than **6***

Soln

(i) $x : y \geq 3 : 1 \;\; \Rightarrow \frac{x}{y} \geq \frac{3}{1} \;\; \Rightarrow \boldsymbol{x \geq 3y}$

(ii) $x : y \leq 4 : 1 \;\; \Rightarrow \frac{x}{y} \leq \frac{4}{1} \;\; \Rightarrow \boldsymbol{x \leq 4y}$

(iii) $x \geq \frac{2}{5}(x + y) \;\; \Rightarrow \;\; \boldsymbol{3x \geq 2y}$

2. *A furniture company has **Shs 120,000** to invest in making tables and chairs**.** It costs **Shs 20,000** to make each table and **Shs 12,000** to make each chair**.** The company has a storage space of at least **8** items altogether**.** Each table yields a profit of **Shs 80,000** and each chair a profit of **Shs 45,000.***

(i) *Write down four inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *List the possible combination of tables and chairs the company can make*

(iv) *Find how many tables and chairs should be made so as to maximize profit and calculate this maximum profit*

Soln

(i) *If **x** = No of tables made and **y** = No of chairs made*

$20{,}000x + 12{,}000y \leq 120{,}000 \Rightarrow \mathbf{5x + 3y \leq 30}$ **- - - - - - (i)**

$\mathbf{x + y \geq 8}$ **- - - - - - (ii)** $\mathbf{x \geq 0}$ **- - - - - - (iii)** $\mathbf{y \geq 0}$ **- - - - - - (iv)**

(iii) *Feasible solution: (0,8), (0,9), (0,10), (1, 7), (1, 8), (2, 6) and (3, 5)*

(iv) *Profit function: f(x, y) = 80,000x + 45,000y* ***"Objective function"***

(x, y)	*(0, 8)*	*(0, 10)*	*(3, 5)*
f(x, y)	*360,000*	*450,000*	*465,000*

$\therefore$ *Max profit is **Shs 465,000,** this includes **3** tables and **5** chairs*

3. *A student has **52** minutes to do a test containing **10** short questions and **5** essay questions. Each short question carries **4** marks and takes **2** minutes to be answered. Each essay question carries **12** marks and takes **8** minutes to be answered. The student knows all the answers to get full marks on the questions he attempts.*

(i) *Write down five inequalities representing the above information*

(ii) *Draw the feasible region that represents this problem*

(iii) *Find how many questions of each type the student should attempt so as to gain maximum marks and find this maximum mark the student can score*

Soln

(i) *If **x** = No of attempted short questions and **y** = No of attempted essay questions*

$\mathbf{x \leq 10}$ **- - - - - - (i)** $\mathbf{y \leq 5}$ **- - - - - - (ii)**

$2x + 8y \leq 52 \Rightarrow \mathbf{x + 4y \leq 26}$ **- - - - - - (iii)**

$\mathbf{x \geq 0}$ **- - - - - - (iv)** $\mathbf{y \geq 0}$ **- - - - - - (v)**

(iii) *Score function: f(x, y) = 4x + 12y* ***"Objective function"***

(x, y)	*(0, 0)*	*(0, 5)*	*(6, 5)*	*(10, 0)*	*(10, 4)*
f(x, y)	*0*	*60*	*84*	*40*	*88*

$\therefore$ *Max score is **88,** this includes **10** short and **4** essay questions*

4. *The area of a parking lot is* $\mathbf{360m^2}$. *Each van requires* $\mathbf{24m^2}$ *of space to park and each bus requires* $\mathbf{48m^2}$. *Not more than* ***12*** *vehicles are allowed to park at a time. Also there must be at least as many vans as buses. If the parking charge for a van is* ***\$1·5*** *and for a bus is* ***\$3·5,***

(i) *Write down five inequalities representing the above information*

(ii) *Draw the feasible region that represents this problem*

(iii) *Find how many vehicles of each type that should be parked so as to obtain maximum income and find this maximum income*

(iv) *If the new charge structure for a van is* ***\$2*** *and for a bus is* ***\$3·5,*** *find how many vehicles of each type that should be parked now so as to obtain maximum income and find this maximum income*

Soln

(i) *If* ***x*** *= No of parked vans and* ***y*** *= No of parked buses*

$$24x + 48y \geq 360 \Rightarrow \mathbf{x + 2y \leq 15} \text{ - - - - - - } \mathbf{(i)} \qquad \mathbf{x + y \leq 12} \text{ - - - - - - } \mathbf{(ii)}$$

$$x : y \geq 1 : 1 \Rightarrow \frac{x}{y} \geq \frac{1}{1} \Rightarrow \mathbf{x \geq y} \text{ - - - } \mathbf{(iii)} \quad \mathbf{x \geq 0} \text{ - - - } \mathbf{(iv)} \quad \mathbf{y \geq 0} \text{ - - - } \mathbf{(v)}$$

(iii) *Income function:* $f(x, y) = 1{\cdot}5x + 3{\cdot}5y$ ***"Objective function"***

(x, y)	*(0, 0)*	*(12, 0)*	*(9, 3)*	*(5, 5)*
f(x, y)	*0*	*18*	*24*	*25*

$\therefore$ *Max income is* ***\$25,*** *this includes* ***5*** *vans and* ***5*** *buses*

(iv) *New income function:* $f(x, y) = 2x + 3{\cdot}5y$ ***"Objective function"***

(x, y)	*(0, 0)*	*(12, 0)*	*(9, 3)*	*(5, 5)*
f(x, y)	*0*	*24*	*28·5*	*27·5*

$\therefore$ *Max income is* ***\$28·5,*** *this includes* ***9*** *vans and* ***3*** *buses*

***5.** A carpenter wishes to make tables and chairs. He can make a maximum of **8** tables or a maximum of **6** chairs per week. Each table requires **5** hours to make and can be sold for a profit of **$25.** Each chair requires **10** hours to make and can be sold for a profit of **$50.** The carpenter only has **70** hours of labour time available per week.*

***(i)** Write down five inequalities representing the above information*

***(ii)** Plot these inequalities on the same axes and shade out the unwanted regions*

***(iii)** Find all the possible combination of tables and chairs that should be made each week so as to maximize profit and find this maximum profit*

Soln

***(i)** If **x** = No of tables made and **y** = No of chairs made*

$x \leq 8$ - - - - **(i)** $y \leq 6$ - - - **(ii)** $5x + 10y \leq 70 \Rightarrow x + 2y \leq 14$ - - - **(iii)**

$x \geq 0$ - - - - - - **(iv)** $y \geq 0$ - - - - - - **(v)**

(iv)** Profit function: $f(x, y) = 25x + 50y$ **"Objective function"

(x, y)	(0, 0)	(0, 6)	(8, 0)	(8, 3)	(2, 6)
f(x, y)	0	300	200	350	350

Since the maximum value is at two vertices, then the optimal solution is on the line segment joining them

$\therefore$ *Max profit is **$350,** this includes **(2,6), (4,5), (6,4)** and **(8, 3)***

***6.** A school wishes to use buses and coasters to transport at least **336** students. Each bus can carry **56** students at a cost of **Shs 50,000.** Each coaster can carry **28** students at a cost of **Shs 25,000.** The school has **10** drivers available and must use at least **4** coasters.*

***(i)** Write down three inequalities representing the above information*

***(ii)** Draw the feasible region that represents this problem*

***(iii)** List the possible combination of buses and coasters that the school can use*

(iv) *Find all the possible combination of buses and coasters the school should use so as to minimize the transport cost and find this minimum transport cost*

(v) *Find how many vehicles of each type the school should use so as to maximize the number of students transported and find this greatest number of students the school can transport*

(i) *If* $\boldsymbol{x}$ *= No of used buses and* $\boldsymbol{y}$ *= No of used coasters*

$56x + 28y \geq 336 \Rightarrow \mathbf{2x + y \geq 12}$ - - - - - - ***(i)***

$\mathbf{x + y \leq 10}$ - - - - - - ***(ii)*** $\quad$ $\mathbf{y \geq 4}$ - - - - - - ***(iii)*** $\quad$ $\mathbf{x \geq 0}$ - - - - - - ***(iv)***

(iii) *Feasible solution*: ***(2, 8), (3, 6), (3,7) (4, 4), (4, 5), (4,6) (5,4), (5, 5)*** *and* ***(6, 4)***

(iv) *Cost function*: $f(x, y) = 50{,}000x + 25{,}000y$ ***"Objective function"***

(x, y)	(2, 8)	(4, 4)	(6, 4)
f(x, y)	300,000	300,000	400,000

Since the minimum value is at two vertices, then the optimal solution is on the line segment joining them

$\therefore$ *Min cost is* ***Shs300,000,*** *this includes* ***(2,8), (3, 6)*** *and* ***(4, 4)***

(v) *Function for No of students*: $f(x, y) = 56x + 28y$ ***"Objective function"***

(x, y)	(2,8)	(4,4)	(6, 4)
f(x, y)	336	336	448

$\therefore$ *Max No of students is* ***448,*** *this includes* ***6*** *buses and* ***4*** *coasters*

7. *A soccer club is to invite players for a soccer training camp. It costs* ***Shs 15,000*** *for each senior player and* ***Shs 6,000*** *for each promoted player. The club only has* ***Shs 675,000*** *to invest in the camp. The club needs at least* ***10*** *more senior players that those promoted and a minimum of* ***20*** *senior players.*

(i) *Write down four inequalities representing the above information*

(ii) *Draw suitable graphs to show the feasible region*

(iii) *Find how many players of each kind the club should invite so as to maximize the number of players at the camp and calculate this maximum number*

Soln

(i) *If* $\boldsymbol{x}$ *= No of invited senior players and* $\boldsymbol{y}$ *= No of invited promoted players*

$15{,}000x + 6.000y \leq 675{,}000 \Rightarrow \mathbf{5x + 2y \leq 225}$ **- - - - - - (i)**

$\mathbf{x - y \geq 10}$ **- - - - - - (ii)** $\mathbf{x \geq 20}$ **- - - - - - (iii)** $\mathbf{y \geq 0}$ **- - - - - - (iv)**

(*Since* $\mathbf{x \geq 20}$***,*** *then* $\mathbf{x}$ *can't be negative thus* $\mathbf{x \geq 0}$ *is not necessary****)***

(iii) *Function for number of player****:*** $f(x, y) = x + y$ ***"Objective function"***

(x, y)	*(20, 0)*	*(20, 10)*	*(35, 25)*	*(45, 0)*
f(x, y)	*20*	*30*	*60*	*45*

$\therefore$ *Max No of players is* ***60,*** *this includes* ***35*** *seniors and* ***25*** *promoted players*

8. *A factory produces shirts and jackets. At least twice as many shirts as jackets are needed. It costs* ***Shs 9,000*** *and takes* ***15*** *minutes to produce a shirt. It costs* ***Shs 4,500*** *and takes* ***30*** *minutes to produce a jacket. The factory operates for at least* ***5*** *hours and only has* ***Shs 450,000*** *to produce these items per day. If it sells each shirt for* ***Shs 12,000*** *and each jacket for* ***Shs 7,000,***

(i) *Write down five inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *Find how many shirts and jackets should be produced per day in order to maximize profit and calculate this maximum profit*

Soln

(i) *If* $\boldsymbol{x}$ *= No of shirts produced and* $\boldsymbol{y}$ *= No of jackets produced*

$x : y \geq 2 : 1 \Rightarrow \frac{x}{y} \geq \frac{2}{1} \Rightarrow \mathbf{x \geq 2y}$ **- - - - - (i)**

$9{,}000x + 4{,}500y \leq 450{,}000 \Rightarrow \mathbf{2x + y \leq 100}$ **- - - - - - (ii)**

$15x + 30y \geq 5(60) \Rightarrow \mathbf{x + 2y \geq 20}$ **- - - - - - (iii)**

$\mathbf{x \geq 0}$ **- - - - - - (iv)** $\mathbf{y \geq 0}$ **- - - - - - (v)**

(iii) *Profit of shirt = 12,000 – 9,000 = 3,000*

Profit of jacket = 7,000 – 4,500 = 2,500

$\Rightarrow f(x, y) = 3{,}000x + 2{,}500y$ ***"Objective function"***

(x, y)	*(10, 5)*	*(20, 0)*	*(50, 0)*	*(40, 20)*
f(x, y)	*42,500*	*60,000*	*150,000*	*170,000*

$\therefore$ *Max profit is* ***Shs 170,000,*** *this includes* ***40*** *shirts and* ***20*** *jackets*

9. *A farmer plans to plant a garden of palm and pine trees. Each palm tree needs* ***30*** *litres of water per day and each pine needs* ***15*** *litres per day. The farmer only has* ***2,100*** *litres of water available per day. He needs to plant at least* ***40,*** *but not more than* ***60*** *pines. He also decides that at least a third of the trees should be palm. He makes a profit of* ***$70*** *on each palm and* ***$25*** *on each pine that he plans.*

(i) *Write down three inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *Find how many trees of each type the farmer should plant to maximize profits and calculate this maximum profit*

Soln

(i) *If* ***x*** *= No of palm trees planted and* ***y*** *= No of pine trees planted*

$50x + 25y \leq 4000 \Rightarrow \mathbf{2x + y \leq 160}$ - - - - - - ***(i)*** $\mathbf{40 \leq y \leq 60}$ - - - - - - ***(ii)***

$x \geq \frac{1}{3}(x + y) \Rightarrow \mathbf{2x \geq y}$ - - - - - ***(iii)***

(iii) *Profit function: f(x, y) = 70x + 25y* ***"Objective function"***

(x, y)	*(20, 40)*	*(30, 60)*	*(50, 40)*	*(40, 60)*
f(x, y)	*2,400*	*3,100*	*4,500*	*4,300*

$\therefore$ *Max profit is* ***Shs 4,500,*** *this includes* ***50*** *palm and* ***40*** *pine trees*

EER:

1. *The feasible region of a linear programming problem is represented by**:***

$$x + 2y \geq 10,\ 3x + 4y \leq 24 \text{ and } x \geq 0$$

(i) *Draw the feasible region that represents this problem*
(ii) *List down all the possible solutions over the feasible region.*
(iii) *Show that the minimum value of the function* ***F(x, y) = 25x + 50y*** *occurs at more than two points and find this minimum value*

[Ans: (iii) (0, 5), (2, 4) *and* ***(4, 3)*** *min value is* ***250]***

2. *A soccer club manager has* ***Shs 500*** *million to spend on buying defenders and forward players**.** It costs* ***Shs 30*** *million to buy each defender and* ***Shs 40*** *million for a forward player**.** The manager needs at least* ***13*** *players altogether and a minimum of* ***6*** *players of each kind**.** The wage per week for each defender is* ***Shs 10*** *million and* ***Shs 20*** *million for forwards**.***

(i) *Write down four inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *List the possible number of players of each kind that the manager can buy*

(iv) *Find how many players of each kind the manager should buy so as to minimize the wage bill and calculate this minimum wage bill*

(v) *Find how many players of each kind the manager should buy so as to spend all the money*

[Ans: (iv) 7 *defenders**,** **6** forwards**,** least wage bill is* ***Shs 190m (v) x = 6, y = 8]***

3. *A company wishes to transport at least* ***480*** *parcels using a lorry and a van. A lorry can carry* ***60*** *parcels at a cost of* ***Shs 45,000*** *per trip. A van can carry* ***40*** *parcels at a cost of* ***Shs 30,000*** *per trip. There is* ***Shs 600,000*** *available for transport. The number of trips made by the lorry should not exceed* ***12.*** *Those made by the van should not exceed twice the number of trips made by the lorry. If* ***x*** *and* ***y*** *are the trips made by the lorry and van respectively,*

(i) *Write down six inequalities representing the above information*

(ii) *Draw suitable graphs to show the feasible region*

(iii) *Find all the possible number of trips made by each vehicle so that the transport cost is minimized and find this minimum transport cost*

[Ans: (iii) (8, 0), (4, 6), (6, 3) *and least cost is* ***Shs 360,000]***

4. *An aeroplane can carry a maximum of* ***200*** *passengers. Each executive class ticket yields a profit of* ***$1000*** *and each economy class ticket yields* ***$600.*** *The airline must reserve at least* ***4*** *times as many economy class seats as executive class seats. It must also reserve at least* ***20*** *executive class seats. If* ***x*** *and* ***y*** *represents the number of executive and economy class tickets sold respectively.*

(i) *Write down three inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *Find how many tickets of each type that should be sold so as to maximize profit and find this maximum profit*

[Ans: (iii) *max profit is* ***$136,000,*** *this includes* ***40*** *executive and* ***160*** *eco tickets****]***

5. *The feasible region of a linear programming problem is represented by:*

$$x \le 8,\ y \le 6,\ x + 2y \le 14,\ x \ge 0 \text{ and } y \ge 0$$

(i) *Draw the feasible region that represents this problem*
(ii) *Find all the possible feasible solutions that maximizes the objective function* ***f(x, y) = 15x + 30y*** *and find this maximum value*

[Ans: (ii) (2,6), (4,5), (6,4) *and* ***(8, 3),*** *max value is* ***210]***

6. *The area of a parking lot is* $\boldsymbol{500m^2}$. *Each bus requires* $\boldsymbol{20m^2}$ *of space to park and each car requires* $\boldsymbol{8m^2}$. *Not more than* ***40*** *vehicles are allowed to park at a time. If the parking charge for a bus is* ***$12*** *and that of a car is* ***$8,***

(i) *Write down four inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *Find how many vehicles of each type that should be parked so as to obtain maximum income and find this maximum income*

[Ans: (iii) *max income is* ***$380,*** *this includes* ***15*** *buses and* ***25*** *cars****]***

7. *A transport company has* ***8*** *lorries of* ***8****–tonnes carrying capacity each and* ***5*** *lorries of* ***10****–tonnes capacity each. There are* ***12*** *drivers available. The company was hired to transport* ***480****– tonnes of cement from the factory to the sales point. Each* ***8****–tonne lorry can make* ***6*** *trips a day and each* ***10****–tonne can make* ***4*** *trips a day. The cost of using an* ***8****–tonne lorry and a* ***10****–tonne lorry are* ***Shs 40,000*** *and* ***Shs 60,000*** *respectively.*

(i) *Show that one of the constraints leads to the inequality* $\boldsymbol{6x + 5y \geq 60}$

(ii) *Write down three further inequalities*

(iii) *Draw the feasible region that represents this problem*

(iv) *Find how many* ***8****–tonne and* ***10****–tonne lorries the company should use so as to minimize the transport cost and find this minimum transport cost*

[Ans: (iv) *min cost is* ***Shs500,000,*** *with****8*** *lorries of* ***8****–tonnes and* ***3*** *of* ***10****–tonnes****]***

8. *Find the minimum and maximum values of the objective function* $\boldsymbol{f(x, y) = 40x + 15y}$, *subject to the following constraints:*

$$5x + 4y > 40, \quad x \geq 2y, \quad 8x + 3y \leq 90 \text{ and } y \geq 0$$

[Ans: min = 285, max = 445]

9. *The area of a music show room is* $\boldsymbol{48m^2}$. *Each executive class seat requires* $\boldsymbol{0 \cdot 8m^2}$ *of floor space and each general class seat requires* $\boldsymbol{1 \cdot 2m^2}$ *of floor space.* *Each executive class ticket yields a profit of* ***£90*** *and each general class ticket yields a profit of* ***£60.*** *At least* ***15*** *executive class tickets are needed and at least two–fifth of the tickets should be general. If* ***x*** *and* ***y*** *represents the number of executive and general class tickets sold respectively,*

(i) *Write down three inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *Find how many tickets of each type that should be sold so as to maximize profit and find this maximum profit*

[Ans: (iii) *max profit is* ***£3,900,*** *this includes* ***30*** *executive and* ***20*** *general tickets****]***

10. *A furniture company wishes to make tables and chairs. Each table requires* ***4*** *hours of carpentry and* ***2*** *hours of varnishing and can be sold for a profit of* ***$70.*** *Each chair requires* ***3*** *hours of carpentry and* ***1*** *hour of varnishing and can be sold for a profit of* ***$50.*** *The company has* ***240*** *hours of carpentry time available and* ***100*** *hours of varnishing per week.*

(i) *Write down four inequalities representing the above information*

(ii) *Draw the feasible region that represents this problem*

(iii) *Find how many items of each type that should be made each week so as to maximize profit and calculate this maximum profit*

[Ans: (iii) *max income is* ***$4,100,*** *this includes* ***30*** *tables and* ***40*** *chairs****]***

11. *A man wishes to mix two food brands* ***P*** *and* ***Q*** *to form a diet rich in vitamins.* *Each bag of food* ***P*** *costs* ***$3*** *and contains* ***6*** *units of vitamin* ***A*** *and* ***5*** *units of vitamin* ***B.*** *Each bag of food* ***Q*** *costs* ***$7*** *and contains* ***3*** *units of vitamin* ***A*** *and* ***10*** *units of vitamin* ***B.*** *The diet is required to contain at most* ***24*** *units of vitamin* ***A*** *and at least* ***50*** *units of vitamin* ***B.*** *If* ***x*** *and* ***y*** *are the number of bags of* ***P*** *and* ***Q*** *to be mixed respectively,*

(i) *Write down four inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *Find how many bag of each brand should be mixed so as to minimize the cost of the diet and calculate this minimum cost*

[Ans: (iii) *min cost is* ***$34,*** *this includes* ***2*** *bags of* ***P*** *and* ***4*** *bags of* ***Q]***

12. *A farmer wishes to mix two fertilizer brands* ***P*** *and* ***Q*** *to enrich his garden with nutrients. Each bag of brand* ***P*** *costs* ***$20*** *and contains* ***2·5kg*** *of nitrogen and* ***6kg*** *of phosphate. Each bag of brand* ***Q*** *costs* ***$50*** *and contains* ***5kg*** *of nitrogen and* ***8 kg*** *of phosphate. The garden needs at least* ***25kg*** *of nitrogen and at most* ***48kg*** *of phosphate. If* ***x*** *and* ***y*** *are the number of bags of* ***P*** *and* ***Q*** *to be mixed respectively,*

(i) *Write down four inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *Find how many bag of each brand should be mixed so as to minimize the cost of the nutrient requirement and calculate this minimum cost*

[Ans: (iii) *min cost is* ***$230,*** *this includes* ***4*** *bags of* ***P*** *and* ***3*** *bags of* ***Q]***

13. *The feasible region of a linear programming problem is represented by:*

$$x + y \leq 30, \; 2x + y \leq 40, \; y \geq 5, \; x \geq 4 \text{ and } 2y \geq x$$

(a) *Draw the feasible region that represents this problem*
(b) *Find the maximum value of* ***F*** *on the feasible region, in case where:*
(i) $F = 3x + y$
(ii) $F = x + 3y$ ***[Ans: b(i) 56 (ii) 82]***

14. *A farmer has **50** hectares of land to grow maize and beans**.** Each hectare of maize yields a profit of **$105** and requires **20** litres of herbicides**.** Each hectare of beans yields a profit of **$90** and requires **10** litres of herbicides**.** The farmer has only **800** litres of herbicides available**.** If **x** and **y** are the hectares of land to be planted with maize and beans respectively**,***

(i) *Write down four inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *Find how many hectares of land should be allocated to each crop so as to maximize profit and calculate this maximum profit*

[Ans: (iii)** max profit is **$4950,** this includes **30** maize hectares and **20** of beans **]

15. *A farmer wishes to mix two fertilizer brands **P** and **Q** to enrich his garden with nutrients**.** Each bag of brand **P** contains **2·5kg** of nitrogen**, 1kg** of phosphate**, 3kg** of potash and **4kg** of chlorine**.** Each bag of brand **Q** and contains **3·5kg** of nitrogen**, 2kg** of phosphate**, 1·5kg** of potash and **5kg** of chlorine**.** The garden needs at least **24kg** of phosphate**,** at least **27kg** of potash and at most **78kg** of chlorine**.** If **x** and **y** are the number of bags of **P** and **Q** to be mixed respectively**,***

(i) *Write down three inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *Find how many bag of each brand should be mixed so as to minimize the amount of nitrogen added in the garden and find this minimum amount of nitrogen added*

[Ans: (iii)** min nitrogen is **45kg,** this includes **4** bags of **P** and **10** bags of **Q]

16. *In a sugar factory, electrical and manual packing machines are to be used. An electrical machine packs* ***300*** *bags per day and a manual one packs* ***250*** *bags per day. An electrical machine requires* ***3*** *workers where as a manual one requires* ***7.*** *At least* ***35*** *workers need to be used and the number of bags packed per day should not exceed* ***3000.*** *If* **x** *and* **y** *represents the number of electrical and manual machines used respectively.*

(i) *Write down four inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *If the cost of running an electrical machine is* ***Shs 6000*** *per day and a manual one is* ***Shs 8000,*** *find how many machines of each type that should be used so as to minimize the cost per day and calculate this minimum cost*

[Ans: (iii) (8, 0), (4, 6), (6, 3) *and least cost is* ***Shs 360,000]***

17. *The party organizing committee has* ***Shs450,000*** *available to spend on buying beer and soda. At least twice as many crates of beer as crates of soda are needed. Each crate of beer contains* ***25*** *bottles and costs* ***Shs40,000.*** *Each crate of soda contains* ***20*** *bottles and costs* ***Shs15,000.*** *More than* ***200*** *bottles of beer and soda are needed.*

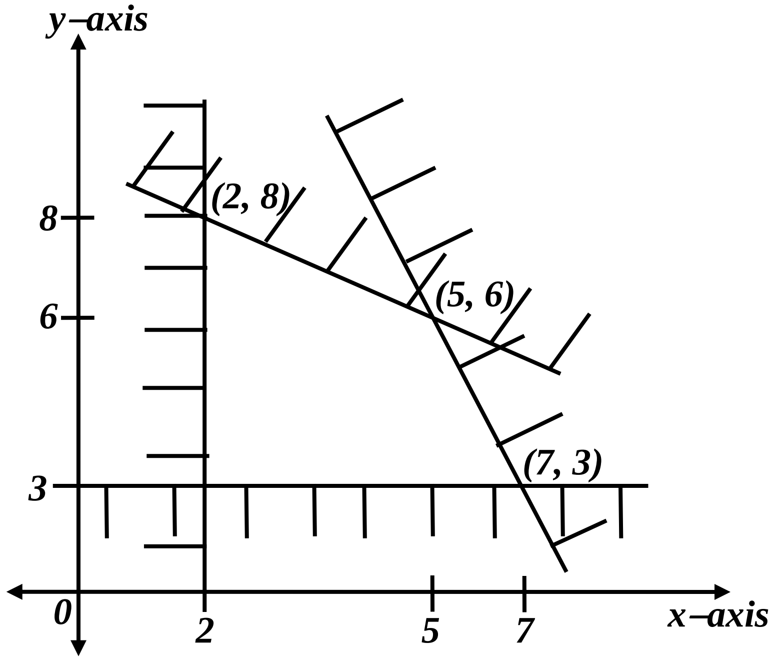
(i) *Write down five inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *Find how many crates of beer and soda that should be bought so as to minimize the cost of drinks and calculate this minimum cost*

[Ans: (iii) *min cost is* ***Shs285,000,*** *this includes* ***6*** *beer crates and* ***3*** *soda crates]*

18. *The graph below shows the feasible region of a linear programming problem:*



(i) *Write down four inequalities representing the feasible region*

(ii) *List down all the possible solutions over the feasible region*

(iii) *Find the maximum value of* $F = 3x + 4y$ *on the feasible region*

(iv) *Calculate the area of the feasible region*

[Ans: (iii) *max value is* ***39*** ***(iv) 15squnits]***

19. *A school Head teacher is to admit* **x** *number of male students and* **y** *female students. The students' admissions are subject to the following conditions:*

$$x \geq 10, \quad y \geq 15, \quad 2y + x \leq 40 \text{ and } y \geq x - 5$$

(i) *Draw the feasible region that represents this problem*
(use a scale of ***2cm:10 units*** *on both axes)*
(ii) *The school fees for a male student is* ***$500*** *and that of a female student is* ***$400.*** *Find how many students of each kind that he should admit in order to obtain maximum income and calculate this maximum income*

[Ans: (iii) *max income is* ***$25,000,*** *this includes* ***30*** *males and* ***25*** *females]*

3. *A school is to hire buses and coasters to transport at least* ***384*** *students****.*** *Each bus can carry* ***64*** *students and a coaster carries* ***48.*** *The school can hire not more than* ***7*** *vehicles****.*** *It costs* ***Shs 75,000*** *to hire a bus and* ***Shs 45,000*** *for a coaster****.***

(i) *Write down four inequalities representing the above information*

(ii) *Plot these inequalities on the same axes and shade out the unwanted regions*

(iii) *Find how many vehicles of each type that should be hired so as to minimize the transport cost and calculate this minimum transport cost*

(i) *If* ***x*** *= No of hired buses and* ***y*** *= No of hired coasters*

$64x + 48y \geq 384 \Rightarrow \mathbf{4x + 3y \geq 24}$ - - - - - - ***(i)***

$\mathbf{x + y \leq 7}$ - - - - - - ***(ii)*** $\mathbf{x \geq 0}$ - - - - - - ***(iii)*** $\mathbf{y \geq 0}$ - - - - - - ***(iv)***

(iii) *Cost function****:*** $f(x, y) = 75{,}000x + 45{,}000y$ ***"Objective function"***

(x, y)	*(3,4)*	*(6,0)*	*(7, 0)*
f(x, y)	*405,000*	*450,000*	*525,000*

$\therefore$ *Min cost is* ***Shs 405,000,*** *this includes* ***3*** *buses and* ***4*** *coasters*

(i) *Write down three inequalities representing the above information*

(ii) *Draw suitable graphs to show the feasible region*

(iii) *Find the number of vehicles of each type that should be used so as to minimize the transport cost and find this minimum transport cost*

[Ans: (iii) (8, 0), (4, 6), (6, 3) *and least cost is* ***Shs 360,000]***

(a) Find how many days of each type of vehicle they should use to
   (i) Minimize the cost of the tour
   (ii) Maximize the distance travelled ***(4 marks)***

1. *A test has* ***6*** *essay questions and* ***25*** *short questions****.*** *A student has only* ***90*** *minutes to do the test****.*** *Each essay question takes* ***10*** *minutes to answer and carries* ***20*** *marks****.*** *Each short question takes* ***2*** *minutes to answer and carries* ***5*** *marks****.*** *At least* ***3*** *essay questions and at least* ***10*** *short questions must be done****.*** *The student knows the material well enough to get full marks on all the questions he attempts****.***

(i) *Write down five inequalities representing the above information*

(ii) *Draw the feasible region that represents this problem*

(iii) *Find how many questions of each type the student should answer so as to score highly and find this highest mark the student can score*